QMM\_Integer\_Assignment

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library(lpSolve)  
  
# Creating a matrix for days and required workers  
DAY\_AND\_WORKERS<- matrix(c("Sunday","Monday","Tuesday","Wednesday","Thursday","Friday","Saturday",18,27,22,26,25,21,19),ncol = 2, byrow = F)   
colnames(DAY\_AND\_WORKERS) = c("Day","Workers\_Required")  
  
# Converting to a table for display  
as.table(DAY\_AND\_WORKERS)

## Day Workers\_Required  
## A Sunday 18   
## B Monday 27   
## C Tuesday 22   
## D Wednesday 26   
## E Thursday 25   
## F Friday 21   
## G Saturday 19

Package handlers at AP are guaranteed a five-day work week with two consecutive days off. The base wage for the handlers is $750 per week. Workers working on Saturday or Sunday receive an additional $25 per day. The possible shifts and salaries for package handlers are:

# Making a table for shifts, days off, and wages  
Shift\_DaysOff\_Wage <- matrix(c(1,2,3,4,5,6,7,"Sunday and Monday","Monday and Tuesday","Tuesday and Wednesday","Wednesday and Thursday","Thursday and Friday","Friday and Saturday","Saturday and Sunday","$775","$800","$800","$800","$800","$775","$750"), ncol = 3, byrow = F)  
  
# Adding column names  
colnames(Shift\_DaysOff\_Wage) <- c("Shift", "Days\_Off", "Wage")  
  
# Converting to a table for display  
as.table(Shift\_DaysOff\_Wage)

## Shift Days\_Off Wage  
## A 1 Sunday and Monday $775  
## B 2 Monday and Tuesday $800  
## C 3 Tuesday and Wednesday $800  
## D 4 Wednesday and Thursday $800  
## E 5 Thursday and Friday $800  
## F 6 Friday and Saturday $775  
## G 7 Saturday and Sunday $750

Question: The manager wants to keep the total wage expenses as low as possible while ensuring that there are sufficient number of workers available each day. Formulate and solve the problem. What was the total cost? How many workers are available each day?

Solution: ##Formulate and solve LP problem

#Objective Function: We aim to minimize the total cost, which is represented by the objective function:

Z = 775X\_1 + 800X\_2 + 800X\_3 + 800X\_4 + 800X\_5 + 775X\_6 + 750X\_7

Where: (X1, X2, X3, X4, X5, X6, X7) are the number of workers under specific shift schedules with corresponding salaries.

#Constraints: We have several constraints to ensure there are sufficient workers available each day:

1. Availability on Sunday: 0X\_1 + 1X\_2 + 1X\_3 + 1X\_4 + 1X\_5 + 1X\_6 + 0X\_7
2. Availability on Monday: 0X\_1 + 0X\_2 + 1X\_3 + 1X\_4 + 1X\_5 + 1X\_6 + 1X\_7
3. Availability on Tuesday: 1X\_1 + 0X\_2 + 0X\_3 + 1X\_4 + 1X\_5 + 1X\_6 + 1X\_7
4. Availability on Wednesday: 1X\_1 + 1X\_2 + 0X\_3 + 0X\_4 + 1X\_5 + 1X\_6 + 1X\_7
5. Availability on Thursday: 1X\_1 + 1X\_2 + 1X\_3 + 0X\_4 + 0X\_5 + 1X\_6 + 1X\_7
6. Availability on Friday: 1X\_1 + 1X\_2 + 1X\_3 + 1X\_4 + 0X\_5 + 0X\_6 + 1X\_7
7. Availability on Saturday: 1X\_1 + 1X\_2 + 1X\_3 + 1X\_4 + 1X\_5 + 0X\_6 + 0X\_7

#Non-Negative Constraints: To ensure a realistic solution, we have non-negative constraints for each variable: [ X\_{1:7} ]

These constraints collectively represent the optimization problem you are solving—minimizing the total cost while meeting the daily worker availability requirements.

Let’s solve the problem using “R”, and by using above mentioned library “lpsolve”. Since, this problem needs to be solved by integer programming, we need to use the function int.vec, which says that all the variables should be of integers and not fractions.

# Define the objective function coefficients  
obj.f <- c(775, 800, 800, 800, 800, 775, 750)  
  
# Defining the constraints matrix  
const <- matrix(c(0, 1, 1, 1, 1, 1, 0,  
 0, 0, 1, 1, 1, 1, 1,  
 1, 0, 0, 1, 1, 1, 1,  
 1, 1, 0, 0, 1, 1, 1,  
 1, 1, 1, 0, 0, 1, 1,  
 1, 1, 1, 1, 0, 0, 1,  
 1, 1, 1, 1, 1, 0, 0), nrow = 7, byrow = TRUE)  
  
const

## [,1] [,2] [,3] [,4] [,5] [,6] [,7]  
## [1,] 0 1 1 1 1 1 0  
## [2,] 0 0 1 1 1 1 1  
## [3,] 1 0 0 1 1 1 1  
## [4,] 1 1 0 0 1 1 1  
## [5,] 1 1 1 0 0 1 1  
## [6,] 1 1 1 1 0 0 1  
## [7,] 1 1 1 1 1 0 0

row.signs <- rep(">=", 7)  
  
# Defining the right-hand side of constraints  
row.rhs <- c(18, 27, 22, 26, 25, 21, 19)  
integer.program\_cost<-lp("min",obj.f,const,row.signs,row.rhs,int.vec=1:9)  
integer.program\_cost

## Success: the objective function is 25675

# Displaying the number of workers for each shift  
  
integer.program\_cost$solution#This show how many workers are corresponding to each shift on respective days

## [1] 2 4 5 0 8 1 13

The weekly salary total is $25,675.

These represent the total employees, along with their salaries and shift schedules, as shown below:

#By using the variables from the lp model   
Shift\_table = matrix(c(0,4,5,0,8,1,0,0,0,5,0,8,1,13,2,0,0,0,8,1,13,2,4,0,0,8,1,13,2,4,5,0,0,1,13,2,3,4,0,0,0,13,2,4,5,0,8,0,0),ncol=7,byrow=TRUE)  
  
colnames(Shift\_table) = c("Shift1", "Shift2", "Shift3", "Shift4", "Shift5", "Shift6", "Shift7")  
  
row.names(Shift\_table) = c('Sunday', 'Monday', 'Tuesday','Wednesday','Thursday','Friday','Saturday')  
  
print(Shift\_table)

## Shift1 Shift2 Shift3 Shift4 Shift5 Shift6 Shift7  
## Sunday 0 4 5 0 8 1 0  
## Monday 0 0 5 0 8 1 13  
## Tuesday 2 0 0 0 8 1 13  
## Wednesday 2 4 0 0 8 1 13  
## Thursday 2 4 5 0 0 1 13  
## Friday 2 3 4 0 0 0 13  
## Saturday 2 4 5 0 8 0 0

The Shift\_table below illustrates the number of available employees for each day, strategically arranged to minimize overall wage expenses.

# Showing the sum of workers present for each day.  
rowSums(Shift\_table)

## Sunday Monday Tuesday Wednesday Thursday Friday Saturday   
## 18 27 24 28 25 22 19